Risk Taking by Entrepreneurs

By Author Undisclosed

Entrepreneurs bear substantial risk, but empirical evidence shows no sign of a positive premium. This paper develops a theory of endogenous entrepreneurial risk taking that explains why self-financed entrepreneurs may find it optimal to invest in risky projects offering no risk premium. Consistently with empirical evidence, the model predicts that poorer entrepreneurs are more likely to undertake risky projects. It also finds that incentives for risk taking are stronger when agents are impatient.

JEL: E21, D21, D92, J23, L25

Keywords: entrepreneurship, risk choice, borrowing constraints, firm dynamics

Entrepreneurial activity is risky and poorly diversified. Most economic models would suggest that the high degree of entrepreneurial risk should be compensated by a significant premium in returns.[1] Yet empirical evidence finds that a premium to entrepreneurial activity is surprisingly low,[2] which raises a question of why people become entrepreneurs. A number of hypotheses have been offered to answer this question, mostly based on the idea that entrepreneurs have a different set of preferences or beliefs (e.g. risk tolerance or overoptimism). This paper provides an alternative theory of endogenous entrepreneurial

[1]For instance, John Heaton and Deborah Lucas (2001) estimate in a portfolio choice model that entrepreneurial returns should exceed the returns to public equity by at least 10 percent. These calculations assume CRRA utility function with the risk aversion coefficient equal to 2.

[2]Barton H. Hamilton (2000) uses data from Survey of Income and Program Participation and estimates that earnings of the self-employed are smaller on average and have higher variance than earnings of workers. Tobias J. Moskowitz and Anette Vissing-Jorgensen (2002) estimate the return to entrepreneurial investment using data from SCF (Survey of Consumer Finances) and FFA/NIPA(the Flow of Funds Accounts and National Income and Product Accounts) and report that the average return to all private equity is similar to that of the public market equity index.

risk taking that does not rely on individual heterogeneity of preferences or beliefs.

We incorporate endogenous choice of entrepreneurial risk in a simple dynamic occupational choice model. A borrowing constrained agent chooses whether to be a worker or an entrepreneur. The occupational choice is discrete, in the sense that each activity requires full time involvement. A worker receives fixed wage income, while an entrepreneur gets access to an entrepreneurial technology and decides how much to invest in it. Borrowing constraints induce endogenous separation into different occupations: the rich, who have sufficient funds for investment, choose to be entrepreneurs, and the poor prefer to receive fixed pay by becoming workers. Hence, the indivisibility of occupational choice may create a nonconcavity in the agent's value function as a function of wealth: even though agents are risk averse in terms of consumption, they become locally risk loving in terms of wealth close to the occupational switch threshold.[3]

This nonconcavity in the agent's value as a function of wealth creates demand for wealth lotteries. We argue that such lotteries, to some extent, can be achieved by entrepreneurial risk taking. The novel feature of our model is that we allow entrepreneurs to choose a project from a set of alternatives. All available projects offer the same expected return but different variance. For the agent who becomes an entrepreneur, investment in a risky project implements a lottery over future wealth that helps eliminate nonconcavities in the his future value. By undertaking such lottery, the entrepreneur effectively randomizes over his future occupations. The closer the entrepreneur's expected future wealth is to the occupational switch threshold, the more willing the entrepreneur will be to invest in a risky project. Since richer entrepreneurs make larger investments and expect to have more wealth in the future, it is the relatively poor entrepreneurs that decide to take more risk and would be more likely to exit from business in the future. As a result, the model

[3]This idea is related to a classic paper by Milton Friedman and Leonard J. Savage (1948), who argued that risk taking behavior may be the result of non-concavities in the indirect utility function. In our model such non-concavities arise endogenously due to the presence of borrowing constraints and discrete occupational choice.

predicts that survival of entrepreneurial business is positively related to entrepreneurial assets, which is consistent with empirical findings.[4]

We also show that there is an interesting interplay between the occurrence of entrepreneurial risk taking and agent's patience. An agent who chooses to be a worker at the beginning of his life and is accumulating wealth in order to switch to entrepreneurship in the future, faces the following tradeoff: enter soon and optimally choose to invest in a risky project (a "risky entrepreneur") or save more and enter later avoiding this risk (the "safe entrepreneur"). Hence, risky entrepreneurship gives access to attractive entrepreneurial technology at an early stage in life at the cost of switching back to fixed pay in the future in case of project failure. In contrast, safe entrepreneurship requires delaying entry but guarantees that the agent remains as an entrepreneur. Thus the choice between entering as a risky or safe entrepreneur crucially depends on how the agent weighs the immediate benefit of risk taking relative to its future costs. That is why we find that risk taking does not occur if agents are very patient, but is likely to happen otherwise.

As mentioned before, risk taking provides a lottery over future wealth that helps mitigate the nonconcavity in the indirect value function resulting from discrete employment choice. Since in our model we consider the extreme case where risky projects have no additional return, risk taking would not occur if, prior to the occupational choice, the agent could take a fair wealth lottery. In reality, however, risky activity is likely to offer some premium, while fair wealth lotteries are hard to come by. We find that, in most cases, either a small cost of lottery or a small premium to investing in a risky project would make entrepreneurial risk taking more attractive than wealth lotteries. Thus we believe that not allowing for fair wealth lotteries can be hardly viewed as a major limitation of our analysis.

Our paper is directly related to several strands of literature. First, there is the obvious

[4]See, for instance, Douglas Holtz-Eakin et al. (1994)

connection to dynamic models of discrete occupational choice. Following Lucas (1978), the models of this type have been successfully used to address a wide variety of economic questions, such as patterns of wealth distribution (e.g. Vincenzo Quadrini 2000), the role of financial intermediation (e.g. Jeremy Greenwood and Boyan Jovanovic 1990, Radim Bohacek 2006), the effects of changes in the tax or bankruptcy regulation (e.g. Marco Cagetti and Mariacristina DeNardi 2006, Ahmet Akyol and Katrik Athreya 2006) and others. As a contribution to this literature, we argue that dynamic occupational choice models may also contribute to explaining excess entrepreneurial risk taking.

Second, a number of explanations have been offered to justify why entrepreneurs might be willing to undertake relatively risky activity: they might be overoptimistic, derive utility from being their own boss or be less risk averse than the rest of the population. To our knowledge, even though the first two ideas have been expressed by a number of authors (e.g., Moskowitz and Vissing-Jorgensen 2002), they have not been explicitly modeled or quantitatively evaluated. The latter explanation, however, has been recently explored in more details in Robert Cressy (2000) and Valery Polkovnichenko (2003) and dates back to Richard Kihlstrom and Jean-Jacque Laffont (1979). While Kihlstrom and Laffont's theory relies on exogenous differences between the entrepreneur's risk aversion coefficients and those of the rest of the population, the papers by Cressy and Polkovnichenko point out that entrepreneurs might be endogenously less risk averse because they are wealthier or have more human capital. In Cressy (2000) all agents have the same DARA utility, different wealth endowments, and can become entrepreneurs by undertaking a risky project of fixed size. Since all entrepreneurs take the same amount of absolute risk, the agents with more wealth choose to become entrepreneurs. In Polkovnichenko (2003) agents differ in their human capital, which cannot be invested in entrepreneurial firm, but still generates additional income. Thus the agents with more human capital risk smaller fraction of their total wealth by becoming entrepreneurs, and hence they might be willing to choose entrepreneurship even if the associated premium is not very high. In both models, however, entrepreneurs would invest in the least risky available project (conditional on the project's expected return) or, if additional risk is compensated by extra premium, richer entrepreneurs would be more willing to bear extra risk. This is in sharp contrast with the predictions of our model, where the presence of the outside opportunity creates incentives for relatively poor entrepreneurs to undertake more risky activity.

Third, the importance of the option value associated with the possibility of closing down the entrepreneurial business has been well recognized in the literature on bankruptcy protection. Several studies (e.g. Akyol and Athreya 2008, Neus Herranz et. al. 2007) show that limited liability by reducing the cost of going bankrupt may stimulate self-employment and entrepreneurship by providing insurance against bad outcomes. Similarly, in our paper, the option of exiting and becoming a worker also serves as insurance against low realizations of entrepreneurial returns. One difference with this literature is that it typically takes the amount of entrepreneurial risk as given and analyzes the effects of various changes in the exit option, while our paper is concerned with the amount and properties of entrepreneurial risk taking that arises endogenously due to the presence of the outside option.

A recent quantitative work by Claudio Campanale (2008) explores an idea related to ours. He develops an occupational choice model, in which agents do not know *ex ante* their own entrepreneurial skills, but can learn them over time by observing the returns to their activity. Thus the observed volatility of entrepreneurial returns among young firms is large, but, due to selection induced by learning, it falls as entrepreneurs age. In Campanale's model, like in ours, entrepreneurs may exit from business and become workers, which creates an option value and makes entrepreneurs risk tolerant at the moment of entry. The author quantifies the amount of risk premia that would justify entry into entrepreneurship in this environment, and finds that it is still substantially larger than what we see in the data. However, in his model the evolution of the volatility of entrepreneurial returns is not chosen by the entrepreneur, but is governed by the learning process and selection. Endogenizing the amount of risk in a way described in our paper makes entrepreneurial activity more attractive and lowers the required risk premium relative to what Campanale reports.

Fourth, the mechanism of risk taking described in our paper also has a number of implications for firm dynamics. In our model relatively poor entrepreneurs are more likely to become workers in the future. At the same time, due to self-financing, they invest less in their projects than richer entrepreneurs. Correspondingly, the model implies that survival rates of business are positively correlated with business size. Moreover, since agents enter entrepreneurship with relatively low wealth levels, our model also implies that young businesses exhibit lower survival rates, and, conditional on survival, small (younger) firms grow faster than larger (older) ones. All these implications are in line with strong empirical evidence from the literature on firm dynamics (see, e.g. David S. Evans 1987, Timothy Dunne, Mark J. Roberts and Larry Samuelson 1989 and Steven Davis and John C. Haltiwanger 1992). Hence, the mechanism of risk taking studied in this paper can also be viewed as a contribution to the theoretical literature explaining the empirical regularities on firm dynamics (e.g. Jovanovic 1982, Hugo Hopenhayn 1992, Richard Ericson and Ariel Pakes 1995).

Finally, our work is also related to a large number of papers that, following Rogerson (1988), use lotteries to deal with nonconvexities. For instance, in a recent paper Gian L. Clementi and Hugo Hopenhayn (2006) rely on lotteries over firms' equity to get rid of nonconvexities in the optimal lending contract resulting from the presence of the liquidation option. The authors pointed out that randomizing in this environment might generate predictions for firm dynamics that are consistent with the data. However, in their paper, as well as in many others (e.g. Hansen 1985, Paulson, et. al. 2006), randomization is used merely as a tool to overcome nonconvexities and does not bear much economic meaning. One of the main contributions of our work is that we actually provide a very intuitive interpretation for lotteries – entrepreneurial project risk choice. This enables us to relate the implications of our model not only to the firm dynamics stylized facts, but also to the broad empirical evidence on entrepreneurial risk taking. In addition, by incorporating the project risk choice in an occupational choice model with exogenous borrowing constraints, we are also able to illustrate how lotteries can be used to relax the borrowing constraints when the agents are sufficiently impatient.

The paper is organized as follows. Section 2 develops a three-period example illustrating the main idea of our paper. Section 3 extends this example to an infinite horizon model and analyzes the properties of entrepreneurial risk taking. We explore conditions under which risk taking occurs, show that risk taking incentives are related to the agents' patience level and provide benchmark computations to illustrate the quantitative working of our model. Finally, Section 4 summarizes our main results and discusses potential extensions, focusing primarily on relaxing some of the extreme assumptions of our model.

I. A three-period example

Consider an agent who lives for three periods and has to choose an occupation in the first two periods of his life. The agent derives utility from consumption in each period. Preferences over consumption profiles are given by $\sum_{t=0}^{2} \beta^t u(c_t)$, where $\beta > 0$ is the time discount factor and c_t is consumption in period t. The utility function $u(\cdot)$ is defined on $(0, +\infty)$, is concave, strictly increasing, and satisfies Inada conditions.

The agent's income profile depends on the occupational decisions made in the first two periods. If the agent decides to be a worker, he earns fixed wage $\phi > 0$. If the agent chooses to be an entrepreneur, he operates a productive project (described below) which generates a payoff in the following period.[5] A crucial feature of the environment is that the agent can change his occupation from the first to the second period at no cost, i.e. the occupational choice is dynamic.

An entrepreneurial project can be selected from a set of available projects with random payoff Zk, where k is the amount invested. We assume that all projects offer the same expected return $EZ = A$, but different levels of risk. The distribution of a project's rates of return is concentrated in two points, x and y, such that $x \leq y$.[6] If the low return x realizes with probability $1 - p$, the high return y can be expressed as

$$y = x + \frac{A - x}{p} \geq A. \tag{1}$$

Therefore, we may identify every available project by the value of the lower return $x \in (0, A]$ and the probability of the higher return $p \in (0, 1)$. If $x = A$, the project is safe, delivering return A for sure; for all other values of x and p the project is risky. The existence of the riskless project that is not dominated in expected return is obviously an extreme assumption. It is convenient for technical reasons and it helps to emphasize the point that risk taking is not necessarily associated with higher returns.

The agent starts at $t = 0$ with initial wealth w_0. At every period the agent has access to a risk-free saving technology that offers a rate of return r. We assume that $1 + r < A$. This assumption is needed to make entrepreneurial activity attractive (if $1 + r \geq A$, no one would ever become an entrepreneur). In the last period no savings are made and all final wealth is consumed. Borrowing is not allowed, neither for consumption nor for

[5]The assumption that the project pays off in the following period is not important. None of the main predictions of our model would be affected if we instead assumed that entrepreneurial income is realized immediately. By delaying the project's payoffs for one period we capture the idea that by investing in his own firm an entrepreneur typically foregoes some investment opportunities offered by the asset market.

[6]This restriction is without loss of generality. As it is illustrated later, the entrepreneur would always choose to invest in a project with the payoff distribution concentrated in two points, as long as such project is available.

entrepreneurial investment purposes. This is an important assumption of our model, it implies that occupational choice is correlated with wealth: relatively poor agents become workers and the rich become entrepreneurs.[7]

Denote by $\{w_t\}_{t=0}^2$ the agent's wealth profile resulting from his occupation and savings decisions. It is most convenient to first characterize the agent's decision problem in the second period, and then describe his choice in the beginning of life. The value of being a worker at $t = 1$ can be expressed as

$$V_1^W(w_1) = \max_{w_2 \geq 0}\{u(w_1 + \phi - \frac{w_2}{1+r}) + \beta u(w_2)\}. \tag{2}$$

First, notice that by investing in a risky project, entrepreneurs randomize over the next period wealth. Since $u(\cdot)$ is strictly concave and all wealth must be consumed in that period, taking a lottery over w_2 decreases the expected last period's utility. Therefore, the entrepreneur always chooses to operate a safe project at $t = 1$. Since $A > 1 + r$, entrepreneurs do not make any savings in the risk-free asset. Thus the value of being an entrepreneur at $t = 1$ can be expressed as

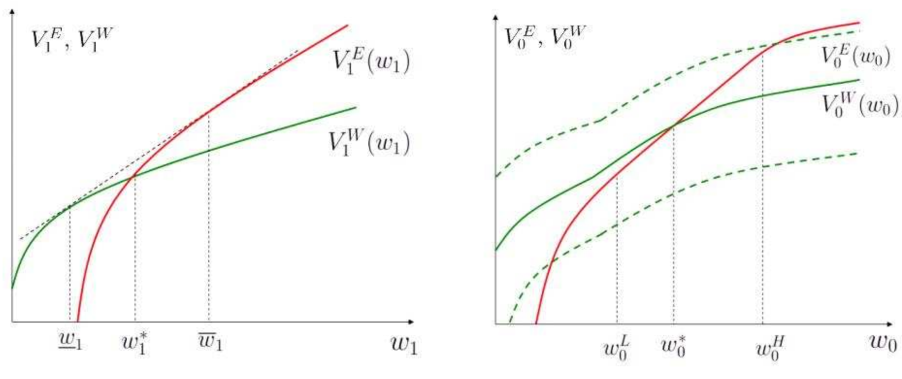
$$V_1^E(w_1) = \max_{w_2 \geq 0}\{u(w_1 - \frac{w_2}{A}) + \beta u(w_2)\}. \tag{3}$$

It is straightforward to verify that the functions$V_1^W(w_1)$ and $V_1^E(w_1)$ are increasing, strictly concave and have a unique intersection at $w_1^* > 0$ such that $V_1^E(w_1) < V_1^W(w_1)$ for all $w_1 < w_1^*$.[8] These value functions are illustrated on the left panel of Figure 1.

[7]A number of empirical studies (e.g. David Blanchflower and Andrew Oswald 1998, William M. Gentry and Glenn R. Hubbard 2004) find evidence consistent with the presence of liquidity constraints on entrepreneurs; while a few recent theoretical papers (e.g. Rui Albuquerque and Hugo Hopenhayn 2004, Clementi and Hopenhayn 2006) illustrate that borrowing constraints may arise endogenously in the presence of various informational problems.

[8]The existence of the intersection follows from $\phi > 0$ and $A > 1 + r$. The former implies that $V_1^W(w_1) > V_1^E(w_1)$ for sufficiently small w_1, from the latter it follows that $V_1^W(w_1) < V_1^E(w_1)$ if w_1 is sufficiently large.

The uniqueness is stipulated by the fact that $A > 1 + r$. Since $u(\cdot)$ satisfies Inada conditions, the



(a) Value functions in period t=1 (b) Value functions in period t=0

FIGURE 1: THE VALUES OF A WORKER AND AN ENTREPRENEUR IN PERIODS $t = 1$ AND $t = 0$.

Denote by $V_1(w_1)$ the value of the agent at $t = 1$, $V_1(w_1) = \max\{V_1^W(w_1), V_1^E(w_1)\}$. A crucial observation is that this value has a kink at w_1^*.

We now turn to the characterization of the agent's decision in period $t = 0$, when entrepreneur solves the following decision problem:

$$
\begin{aligned}
V_0^E(w_0) = \max_{p,x,k_0\geq 0,b_0\geq 0} & \left\{u(w_0 - k_0 - b_0) + \beta[pV_1(w_1^h) + (1-p)V_1(w_1^l)]\right\} \\
\text{s.t. } & w_1^h = yk_0 + (1+r)b_0 \\
& w_2^0 = xk_0 + (1+r)b_0 \\
& py + (1-p)x = A, \quad p \in (0,1), \quad x \in (0, A].
\end{aligned} \tag{4}
$$

Since the continuation value $V_1(w_1)$ is not concave, investing in a risky project, and thus randomizing over the next period's wealth, can be beneficial. Consider an entrepreneur who chooses to invest k_0 in his project and save b_0 in a risk-free bond. If this entrepreneur

decision problems (2) and (3) have interior solutions. Therefore, $V_1^{E'}(w_1) = u'(c_1^E) = \beta A u'(w_1^E)$ and $V_1^{W'}(w_1) = u'(c_1^W) = \beta(1+r)u'(w_1^W)$, where c_1^E and c_1^W are the optimal consumption levels of the entrepreneur and the worker, and w_1^E and w_1^W are the corresponding wealth levels at $t = 2$. Thus $V_1^{E'}(w_1) \leq V_1^{W'}(w_1)$ implies that $V_1^E(w_1) > V_1^W(w_1)$, suggesting that only one intersection at which $V_1^{E'}(w_1) > V_1^{W'}(w_1)$ may exist.

operates a safe project, his next period value would be equal to $V_1(Ak_0 + (1+r)b_0)$. If $Ak_0 + (1+r)b_0$ falls between $\underline{w}_1$ and $\overline{w}_1$ shown on panel (a) of Figure 1, the entrepreneur can increase his expected continuation value by investing in a risky project generating a final wealth that randomizes between points $\underline{w}_1$ and $\overline{w}_1$. This is achieved by setting $xk_0 + (1+r)b_0 = \underline{w}_1$ and $yk_0 + (1+r)b_0 = \overline{w}_1$,[9] thus delivering the expected value $pV_1(\underline{w}_1) + (1-p)V_1(\overline{w}_1) = \widehat{V}_1(Ak_0 + b_0)$, where $\widehat{V}_1(\cdot)$ is the concave envelope of $V_1(\cdot)$. In other words, risk taking allows the entrepreneur to eliminate the kink in his continuation value, which appears due to the possibility of occupational switch in the future.

Given that the entrepreneur chooses the project optimally, the savings and investment decision is found from the following problem:

$$V_0^E(w_0) = \max_{k_0 \geq 0, b_0 \geq 0} \left\{ u(w_0 - k_0 - b_0) + \widehat{V}_1(Ak_0 + (1+r)b_0) \right\}. \tag{5}$$

Now it is apparent that, since $A > 1 + r$, entrepreneurs make no savings in the risk-free asset, i.e. $b_0 = 0$. Concavity of $\widehat{V}_1(w_1)$ implies that optimal investment $k_0(w_0)$ increases with wealth w_0, i.e. richer entrepreneurs operate bigger projects (own larger firms). Since a risky project is chosen if and only if Ak_0 falls in the interval $(\underline{w}_1, \overline{w}_1)$, there exists an interval (w_0^L, w_0^H) such that the entrepreneur at $t = 0$ operates a risky project if and only if his initial wealth $w_0 \in (w_0^L, w_0^H)$. Outside of this interval, entrepreneurs invest only in the safe project and at $t = 1$ either (i) become a worker if $w_0 \leq w_0^L$ or (ii) continue to operate the safe project if $w_0 \geq w_0^H$.

The following proposition summarizes the above discussion.

PROPOSITION 1: *(Entrepreneurial project choice at $t = 0$)*

There exist $0 < w_0^L < w_0^H$ such that, conditional on becoming an entrepreneur at $t = 0$,

[9] The project set is rich enough to guarantee that the risky project with the desirable properties exist for every (k_0, b_0) for which $Ak_0 + (1+r)b_0 \in (\underline{w}_1, \overline{w}_1)$.

the agent with initial wealth w_0

(i) operates a safe project at $t = 0$ *and becomes a worker at* $t = 1$ *if* $w_0 \leq w_0^L$*;*

(ii) operates a risky project at $t = 0$ *and, depending on the project's return becomes either a worker or a safe entrepreneur in period* $t = 1$ *if* $w_0 \in (w_0^L, w_0^H)$*;*

(iii) operates a safe project at $t = 0$ *and continues as a safe entrepreneur at* $t = 1$ *if* $w_0 \geq w_0^H$*.*

Thus far we have argued that the possibility of repeated occupational choice creates incentives for entrepreneurial risk taking. However, we have not yet described how the occupational choice is made at $t = 0$. The value of the worker $V_0^W(w_0)$ at $t = 0$ is given by

$$V_0^W(w_0) = \max_{w_1 \geq 0}\{u(w_0 + \phi - \frac{w_1}{1+r}) + \beta V_1(w_1)\}. \tag{6}$$

It is tedious but straightforward to verify that value functions $V_0^W(w_0)$ and $V_0^E(w_0)$ have a unique intersection. However, it is hard to say where exactly their intersection occurs. As panel (b) of Figure 1 illustrates, $V_0^W(w_0)$ might cross $V_0^E(w_0)$ to the right, to the left or in the middle of the risk taking interval (w_0^L, w_0^H). In the first case there are no initial wealth levels at which an agent chooses to become a risky entrepreneur at $t = 0$. In other words, while there is a nonempty range of wealth levels at which an entrepreneur would take the risky project, an agent might instead prefer to be a worker. In particular, the value of becoming a risky entrepreneur may be dominated by starting as a worker, saving in a risk-free bond and then entering into entrepreneurship without risk.

Figure 2 illustrates how incentives for entrepreneurial risk taking are affected by the parameters of the model when the utility function is logarithmic.[10] The left plot shows

[10]Detailed calculations are available in the online Appendix A.

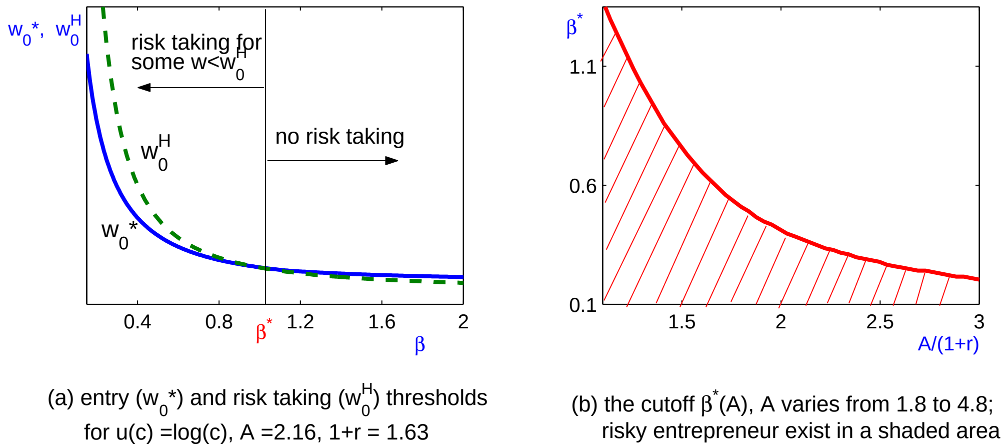


FIGURE 2: EXISTENCE OF RISKY ENTREPRENEURS AT $t = 0$

the upper bound of the risk taking interval w_0^H and the entry threshold w_0^* for different values of β when $A = 2.16$ and $1 + r = 1.63$.[11] We can see that risk-taking occurs only for values of β below 0.97.[12] The right plot shows how this critical value of β varies with A. In this example, risk taking occurs if agents are impatient and the relative return to entrepreneurship is not too high. There is an intuitive explanation: patience favors the alternative of waiting and avoiding risk; high returns to entrepreneurship increase the desired level of investment in this technology and thus the returns to accumulating additional wealth prior to entry.

The following section extends the model considered here to an infinite horizon environment with arbitrary utility function. This extension allows us to provide a sharper characterization of the relationship between the model's parameters and risk taking incentives. Extending the three period model is also useful for other reasons. First, by construction of the three-period example, entrepreneurs can choose to invest in risky projects only once in their lifetime. In contrast, in a many-period setting we can analyze

[11]These values of A and $1 + r$ correspond to the returns obtained in 10 years if the annual returns are 1.08 and 1.05 respectively.

[12]When one period is equal to 10 years, $\beta = 0.97$ corresponds to an annual discount factor of 0.9969. Therefore, the set of parameters, for which the risk taking interval is not empty, is quite reasonable.

whether entrepreneurs would choose to make risky investment repeatedly (i.e. taking small risks over many periods or big risk only once). Second, we can quantify the amount of entrepreneurial risk taking under sensible parameter choice in order to analyze whether the mechanism of entrepreneurial risk taking outlined in this paper can potentially contribute to understanding the excessive entrepreneurial risk documented in the data.

II. Infinite Horizon Occupational Choice Model

In this Section we extend a three-period example analyzed above to a more general infinite horizon setting. We characterize entrepreneurial project choice, establish that there exists a relationship between the agent's patience and incentives to invest in risky projects and, using a series of numerical exercises, argue that the mechanism of entrepreneurial risk taking studied in this paper may have significant quantitative implications.

A. Environment

Consider an infinitely-lived agent who chooses in every period whether to be employed as a worker or manage his own business as an entrepreneur, but cannot participate part time in each. There are no barriers to occupational mobility, i.e. in every period the agent can costlessly switch from being a worker to being an entrepreneur and vice versa.

The agent's preferences are given by $\sum_{t=0}^{\infty} \beta^t u(c_t)$. In addition to the assumptions made about $u(\cdot)$ in the previous section, we also assume here that $u(\cdot)$ is bounded from above.

The agent's income profile is determined by the sequence of occupational decisions. Technology is the same as in the example discussed in the previous section: a worker receives a constant wage ϕ and can save in a risk-free asset with the rate of return r, while an entrepreneur has access to a set of projects $\Omega(A)$ which pay a random return Z at the beginning of the following period (i.e. an entrepreneur investing k in his project

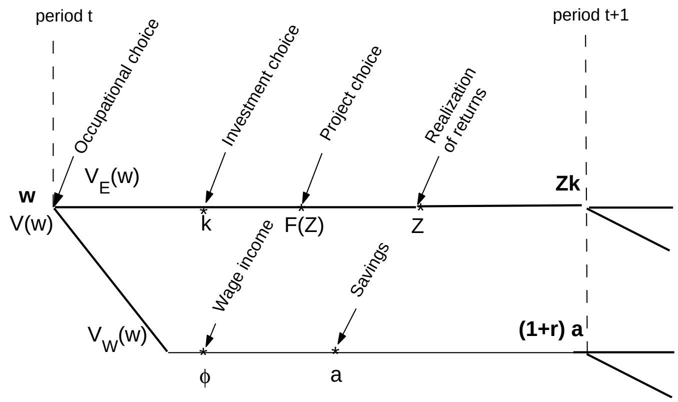


Figure 3: Timeline of the agent's decision

obtains income Zk in the following period[13]). Available projects offer the same expected return A but different degrees of risk. Assume that the set of projects $\Omega(A)$ contains all possible mean preserving spreads of a fully safe project with the rate of return A, i.e. $\Omega(A) = \{P : E(Z) = A\}$. We also assume that $A > 1 + r$ and $\beta A > 1$.

We set up the agent's decision problem recursively. The sequence of decisions that are made within each period is illustrated on Figure 3: the agent enters the period with wealth w, makes an occupational decision, then chooses the investment level and the type of project (as an entrepreneur) or the amount of savings in a risk-free asset (as a worker).[14] Denote the values from being a worker and an entrepreneur in the current period by $V_W(w)$ and $V_E(w)$ respectively. Then the value of the agent $V(w)$ in the beginning of the period, prior to choosing occupation, is given by

$$V(w) = \max\{V_E(w), V_W(w)\}. \tag{7}$$

[13]The assumption of linear entrepreneurial technology is not crucial for any of our results, it is adopted in order to simplify the exposition.

[14]Recall that in the previous section we have argued that the entrepreneur would never save in a risk-free asset since entrepreneurial technology offers higher return and the value of the entrepreneur is linear in all risk taking intervals, implying that no insurance is needed to complement the risky investment. Obviously, the same result also obtains in an infinite horizon model. Thus we right away assume that the entrepreneur does not make use of the available saving technology.

The value of an entrepreneur is

$$V_E(w) = \max_{k \geq 0, P \in \Omega(A)} \{u(w-k) + \beta E V(Zk)\}, \tag{8}$$

and the value of a worker is given by

$$V_W(w) = \max_{a \geq 0} \{u(w + \phi - a) + \beta V((1+r)a)\}. \tag{9}$$

Note that both a worker and an entrepreneur continue with the value $V(w)$ in the following period because the change of occupation can happen any time.

Equations (7)-(9) fully describe the decision problem of the agent in this simple discrete occupational choice problem. Due to the recursive structure of the problem, the solution methodology has a lot in common with the analysis of the agent's decision in the first period of the three-period example studied earlier. In the rest of this section we generalize this approach to an infinite horizon model and sketch the sequence of steps necessary to understand the properties of entrepreneurial risk taking.

B. Occupational choice

Since borrowing is not allowed, an agent with little wealth chooses to become a worker and receive a fixed wage income, while a richer agent decides to give up the fixed wage in favor of more profitable investment opportunities by becoming an entrepreneur. Thus $V_W(w)$ and $V_E(w)$ intersect at least once. The following proposition establishes the uniqueness of the cutoff wealth level w_E which determines the agents' occupational choice.

PROPOSITION 2: *(Single crossing)*

There exists $w_E \geq 0$ such that $V_W(w) \geq V_E(w)$ if and only if $w \leq w_E$.

In addition, $V'_W(w_E) < V'_E(w_E)$.

The proof of Proposition 2 is quite technically intense and is available in online Appendix B. In addition to single crossing, Proposition 2 also establishes that a kink necessarily occurs at the cutoff wealth level w_E, i.e. $\max\{V_W(w), V_E(w)\}$ is not concave. This prediction is important for generating entrepreneurial risk taking. It also has a very simple intuitive explanation. Workers obtain smaller returns to saving than entrepreneurs do (recall that $1+r < A$). Therefore a worker's wealth grows at a lower rate than that of an entrepreneur. Thus the continuation value of the worker at the cutoff wealth level w_E is smaller than the next period's expected value of the entrepreneur with the same wealth level. Since $V_W(w_E) = V_E(w_E)$, it means that the worker at w_E must enjoy higher current consumption, which, by the envelope condition, implies that $V_W(w)$ is flatter than $V_E(w)$ at their intersection point.

C. Entrepreneurial project choice

From the decision problem (8) we can see that the choice of the entrepreneur can be separated in two steps: (i) consumption/investment choice and (ii) optimal project choice. We first characterize the choice of the optimal project given the size of entrepreneurial investment, and then describe how the consumption/investment decision is made.

As in the three-period example in the previous section, the entrepreneur uses risky projects in order to eliminate kinks in his continuation value $V(w) = \max\{V_W(w), V_E(w)\}$ illustrated on Figure 4. To see this, consider an entrepreneur who invests k in his business. If the safe project is chosen, next period value is given by $\max\{V_W(Ak), V_E(Ak)\}$. From Proposition 2 we know that $V(Ak)$ definitely has a kink at the crossing point of $V_W(Ak)$ and $V_E(Ak)$. Thus, if the payoff to the safe project Ak falls in the neighborhood of w_E, the entrepreneur would be better off operating a risky project with payoffs concentrated in two points, $\underline{w}$ and $\overline{w}$, where $\underline{w}$ and $\overline{w}$ are the tangent points of $V_W(Ak)$ and $V_E(Ak)$

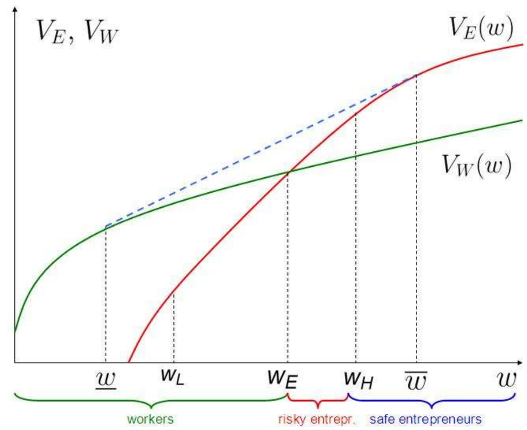


Figure 4: The properties of the agent's value functions $V_E(w)$ and $V_W(w)$

with their common tangent line. The returns offered by the optimal project are found from

$$x(k)k = \underline{w} \quad \text{and} \quad y(k)k = \overline{w}. \tag{10}$$

Since the project must generate expected return A, the probability $p(k)$ of the realization of low return is determined by the condition $p(k)x(k) + (1 - p(k))y(k) = A$, which, in conjunction with (10), implies that

$$p(k) = \frac{Ak - \underline{w}}{\overline{w} - \underline{w}}. \tag{11}$$

By investing in this risky project, the entrepreneur effectively randomizes between the future options of staying in business (with probability $p(w)$) and exiting to become a worker (with probability $1 - p(w)$).

Correspondingly, if the entrepreneur's investment is such that $Ak \in (\underline{w}, \overline{w})$ then his expected next period value becomes equal to $\widehat{V}(Ak) = p(k)V_W(\underline{w}) + (1 - p(k))V_E(\overline{w})$.

In addition, by choice of $\underline{w}$ and $\overline{w}$, the entrepreneur's continuation value is smooth at the boundaries of the risk taking interval.[15] Hence, the possibility of operating risky projects allows an entrepreneur to eliminate all nonconcavities in his continuation value.[16] Note also that a risky entrepreneur would want to use only the projects with returns concentrated in two points. Thus it is sufficient to limit the set of available projects $\Omega(A)$ to $\Omega_2(A) = \{(x, p) : x \in (0, A], p \in (0, 1)\}$, as it was done in the example in the previous section.

We now turn to the entrepreneur's consumption/investment decision. Denote by $\widehat{V}(w)$ the concave envelope of $\max\{V_W(w), V_E(w)\}$. Then entrepreneurial investment $k(w)$ solves the following dynamic problem:

$$V_E(w) = \max_{k \geq 0}\{u(w - k) + \beta\widehat{V}(Ak)\}. \tag{12}$$

Since $\widehat{V}(w)$ is concave, the optimal investment $k(w)$ must be monotone in wealth. This implies that $Ak(w)$ falls into the interval $(\underline{w}, \overline{w})$ if and only if the current wealth of the entrepreneur w belongs to some interval (w_L, w_H). Inside this interval the entrepreneur invests in a risky project and, depending on the realized project's return, may either stay in business or exit to become a worker in the following period. The end points of this interval correspond to the wealth levels at which optimally sized safe projects generate payoffs $\underline{w}$ and $\overline{w}$ respectively, i.e. $Ak(w_L) = \underline{w}$ and $Ak(w_H) = \overline{w}$.

In general, the cutoff wealth level w_E can fall to the left, to the right or inside the risk

[15]Note that, since the continuation value $\widehat{V}(Ak)$ is linear in $(\underline{w}, \overline{w})$, the current value $V_E(w)$ must be linear in (w_L, w_H) and, correspondingly, all the entrepreneurs with current wealth inside (w_L, w_H) must have the same current period consumption. In addition, since $V_W'(\underline{w}) = V_E'(\overline{w})$, the next-period consumption of a risky entrepreneur does not depend on the realization of the project's returns, and only the shape of the future consumption profile will be affected by the outcome of risky investment.

[16]Potentially, in addition to the kink at w_E, other non-concavities can appear in $V(Ak)$. If this happens, all of them can be smoothed out through the optimal project choice. However, notice that single crossing of $V_W(Ak)$ and $V_E(Ak)$ implies that the interval, within which the entrepreneur randomizes between exiting and staying in business in the following period, is unique.

taking interval (w_L, w_H). We will discuss how the model's parameters affect its location below. At this stage, however, it is easy to verify that assumption $\beta A > 1$ guarantees that $w_H < \overline{w}$.[17] Thus all potential entrepreneurs can be separated into three groups in accordance with their current period wealth: (i) the ones with $w \geq w_H$ invest in a safe project and remain in business in the following period, (ii) the ones with $w \in (w_L, w_H)$ invest in a risky project and exit from business in the next period with positive probability and (iii) the ones with $w < w_L$ invest in either safe or risky projects[18] and, independently of the projects' return, exit and become workers in the following period. Hence, our model predicts that entrepreneurial business survival is correlated with the wealth of the business owner: rich entrepreneurs continue operating their firms in the future, and poor entrepreneurs exit from business with positive probability in the following period. This prediction of our model is consistent with the empirical evidence reported by Holtz-Eakin et. al. (1994).[19]. In addition, since entrepreneurial investment is increasing in wealth, the model's predictions are also consistent with the evidence from firm dynamics literature (e.g. Dunne et. al. 1989): smaller firms are more likely to exit and, conditional on survival, have higher growth rates.

Finally, since $w_H < \overline{w}$, the model predicts that an entrepreneur who has invested in a risky project once and was lucky to obtain a high payoff would remain a safe entrepreneur forever.[20] This is a stylized feature of our model driven by the availability of a fully

[17] To see why this is true, observe that a standard recursive argument can be used to show that, for all $w \geq \overline{w}$, $V_E(w)$ coincides with the value $V_E^f(w)$ of the agent who remains a safe entrepreneur forever and found from the programming problem $V_E^f(w) = \max_{k \geq 0}\{u(w-k) + \beta V_E^f(Ak)\}$.

The argument goes as follows. If $V_E(w)$ coincides with the $V_E^f(w)$ when $w \geq \overline{w}$, then $V(Ak) = \max\{V_W(Ak), V_E(Ak)\}$ must be strictly concave for all $w \geq \overline{w}$, which means that no risk taking occurs if $Ak(w) \geq \overline{w}$ or, equivalently, $w \geq w_H$. This implies that $V_E(w) = V_E^f(w)$ for all $w \geq w_H$, and, since $\beta A > 1$, it must be that $w_H < \overline{w}$. This also verifies that $V_E(w) = V_E^f(w)$ indeed holds for $w \geq \overline{w}$.

[18] Since a worker does not have tools for eliminating kinks in his continuation value, the value function $V_W(w)$ is not necessarily concave in $(0, w_E)$. Thus an entrepreneur with $w < w_L$ might choose to invest in a risky project and exit independently of the realization of the project's payoff.

[19] Holtz-Eakin et. al. (1994) find, using individual income tax return data, that the likelyhood of remaining a sole proprietor (business owner) increases if an individual receives inheritance.

[20] This prediction is not driven by the infinite horizon, it would also be obtained in a finite horizon environment. To see this, notice that, by construction of the risk taking interval, the entrepreneur's

safe project and the richness of the available projects' set. In a more realistic setting, entrepreneurial risk taking could occur repeatedly. For instance, it can be easily verified that if the probability of the project's high payoff is bounded from below ($p \geq \underline{p} > 0$), an entrepreneur would take risk in the first few periods in business and, conditional on succeeding in all of them, would continue with the safe project thereafter. Alternatively, if entrepreneurial activity had an uninsured risky component (i.e., a fully safe project were not available), an entrepreneur would reenter into a risk taking interval after a sequence of bad realizations. Note, however, that in either modification the amount of risk taking would still be negatively correlated with wealth (as well as project size) since the the possibility of exiting and becoming a worker provides an option value only for those entrepreneurs who are sufficiently close to the cutoff w_E.

D. *Entry into entrepreneurship and risk taking*

As we noted in the three-period example earlier, the presence of a kink in the entrepreneur's continuation value does not mean that entrepreneurial risk taking necessarily takes place. In particular, it might happen that instead of becoming a risky entrepreneur the agent may prefer to continue as a worker, save and enter into entrepreneurship without risk. Naturally, in a model with long horizon where the agent has more flexibility in choosing the savings profile and the moment of entry into entrepreneurship, such option may become especially valuable. Thus one might expect that in an infinite horizon model incentives for entrepreneurial risk taking might vanish. In this section we show that this would indeed happen if the agent is patient enough. If, on the other hand, the agent is sufficiently impatient, entrepreneurial risk taking occurs and could be quantitatively

continuation value must be strictly concave at the wealth level corresponding to a risky project's payoff. At the same time, the value of the entrepreneur must be linear inside the risk taking interval. Thus an entrepreneur who has already invested in a risky project would make only safe investment for as long as he remains in business, since it is straightforward to verify that he would remain on strictly concave parts of all his future value functions.

significant.

We start by characterizing a worker's saving profile and the decision of whether or not to become an entrepreneur in the future. Not surprisingly, the interplay between the agent's patience parameter β and the risk-free interest rate r plays a crucial role in shaping the worker's wealth dynamics. In the absence of entrepreneurial opportunities, the worker's assets would be strictly increasing over time if and only if $\beta(1+r) > 1$. The option of becoming an entrepreneur stimulates the worker's savings and may induce him to choose an increasing wealth profile even if $\beta(1+r) \leq 1$. However, in the latter case, the entrepreneurial opportunities are attractive only if the worker's initial wealth is sufficiently large so that he can become an entrepreneur soon enough. For lower levels of wealth the agent will rather prefer to remain a worker forever and gradually consume down his initial endowment.[21] The following Lemma summarizes this intuitive argument.[22]

LEMMA 3: *(A worker's wealth dynamics)*

Denote by $a(w)$ the optimal savings decision of a worker.

(i) If $\beta(1+r) > 1$, then $(1+r)a(w) > w$ for all $w \in [0, w_E]$.

(ii) If $\beta(1+r) \leq 1$, then there exists $\widetilde{w} \in [0, w_E]$ such that $(1+r)a(w) < w$ for all $w \in [0, \widetilde{w})$ and $(1+r)a(w) > w$ for all $w \in [\widetilde{w}, w_E]$.

Now consider a worker who is saving in order to become an entrepreneur in the future. It is easy to verify that if such behavior arises for some w (i.e. if $\beta(1+r) > 1$ or $\beta(1+r) \leq 1$ and $\widetilde{w} < w_E$), the agent would never enter into entrepreneurship to the left of the randomization interval.[23] The agent chooses between entering soon as a risky

[21]This characterization of the worker's behavior describes how "a poverty trap" might emerge in an occupational choice model if $\beta(1+r) \leq 1$. Similar properties were established by Francisco Buera (2008) who analyzed an occupational choice model in continuous time.

[22]The proof of Lemma 1 is in online Appendix B.

[23]This would imply that the agent exits for sure in the following period. Lemma 4 in the online appendix formally proves that $w_E > w_L$ in this case.

entrepreneur with relatively low wealth (in which case $w_L < w_E < w_H$) or accumulating wealth and entering later as a safe entrepreneur with relatively high wealth (in which case $w_E \geq w_H$). The tradeoff between these decisions can be summarized as follows. If the agent chooses to enter as a safe entrepreneur, he needs to wait longer till he accumulates the necessary amount of wealth, but upon entering enjoys high entrepreneurial returns for the rest of his life.[24] In contrast, starting as a risky entrepreneur allows the agent to enter earlier, enjoy high entrepreneurial returns at an earlier stage in life, but at the cost of switching back to the less efficient saving technology in the future in case of project failure. Thus the choice between entering as a risky or safe entrepreneur crucially depends on how the agent weighs the immediate benefits of risk taking relative to its future expected costs. The following Proposition establishes that if the agent is sufficiently patient, the latter dominates and entrepreneurial risk taking does not occur.

PROPOSITION 4: *(Patience leads to no risk taking)*
If $\beta \geq \frac{1}{1+r}$, then $w_E > w_H$ and all entrepreneurs invest in safe projects.

Naturally, for less patient agents the option of accumulating wealth in order to enter as a safe entrepreneur in the future is less attractive. The following Proposition establishes that when the time discount factor β is sufficiently close to $1/A$, the agent never delays entry choosing to either remain a worker for the rest of his life or become an entrepreneur immediately. It also turns out that when β is sufficiently close to $1/A$, the poorest entrepreneurs prefer to invest in a risky project. To see the latter point, suppose that risk taking did not occur. Then the two relevant options are to remain forever as a worker or a safe entrepreneur. Consider an agent who is indifferent between these two options.[25] We will argue that this agent should strictly prefer to become a risky entrepreneur. If $1/A \leq \beta < 1/(1+r)$, the worker's consumption declines over time, and the entrepreneur's

[24]Recall that the agents with $w \geq w_H$ remain safe entrepreneurs forever.
[25]Assumptions $\phi > 0$ and $A > 1 + r$ assure that such an agent exists.

consumption (weakly) rises. Since the two options give exactly the same value, it must be that the worker's initial consumption is strictly higher than the entrepreneur's, implying by the envelope condition that the maximum of the two values necessarily has a kink at the cutoff wealth level. If $\beta = 1/A$, the safe entrepreneur's wealth should remain constant over time. This means that the safe entrepreneur with the threshold wealth level, at which the kink occurs, would be strictly better off investing in a risky project (since his continuation value is locally convex around his next period's wealth). By continuity, the same holds if β is sufficiently close to $1/A$. Thus risk taking must occur if the agents are sufficiently impatient. This is formally stated in the Proposition 4 below.

PROPOSITION 5: *(Impatience leads risk taking)*
There exists $\underline{\beta} \in (\frac{1}{A}, \frac{1}{1+r})$ *such that for all* $\beta \in (\frac{1}{A}, \underline{\beta})$ *the risk taking interval is nonempty (i.e.* $w_E < w_H$*) and all entry occurs in period 0 (i.e.* $\widetilde{w} = w_E$*).*

Thus far we have shown that if the agent is patient enough, the option of investing in a risky project is dominated by the opportunity to postpone entry, gradually accumulate wealth while receiving the wage, and then enter without risk. If, on the other hand, the agent is impatient he would never choose to postpone entry into entrepreneurship and, depending on the initial wealth level, he may either remain a worker forever or start right away as a safe or as a risky entrepreneur. The next step would be to investigate whether at some intermediate patience levels the agent can become a risky entrepreneur after having worked as a worker for a while. Though we were not able to show formally that such type of behavior occurs, our quantitative analysis illustrates that this behavior indeed arises for intermediate values β.

Another way to see why the workers saving for future entrepreneurship can benefit from risk taking at the moment of the occupational switch is to look at the shape of their consumption profile and analyze how it is affected by risk taking. In the absence of risk taking, if $\beta(1+r) < 1$ and the agent chooses to start saving for future entrepreneurship, his consumption falls as long as he remains a worker and starts growing only after he becomes a safe entrepreneur (since $\beta A > 1$). Risk taking gives a chance to shorten the period of falling consumption by shortening the period of saving for entrepreneurship. If risk taking is possible, the agent may enter earlier and invest in a risky project at the moment of entry. In case of success, his wealth rises substantially, and he "jumps" right away to the increasing part of his consumption profile (with the obvious benefit of eliminating the inverse hump in it). Thus, in some sense, the risk taking opportunity helps this worker overcome his borrowing constraint. In case of failure, the agent's wealth declines, but his consumption does not – because in a range of lower wealth levels the agent actually enjoys higher instantaneous consumption. The lower is the time discount factor, the faster the worker's consumption falls, and the bigger are the incentives to smooth the U-shaped consumption profile by turning to risk taking.

Understanding how the non-monotonicity of consumption profile creates incentives for entrepreneurial risk taking is also important because it suggests that, for a number of straightforward modifications of our model, risk taking may arise even if $\beta(1+r) \geq 1$. A natural extension of our environment, which has been widely used in the occupational choice literature,[26] would be to incorporate some kind of uninsured risk – for example, by adding shocks to the worker's wage and entrepreneurial returns, or by allowing for random arrival and death of entrepreneurial ideas. Then some realizations of uncertainty would trigger entry into entrepreneurship accompanied by a sudden decline in immediate consumption. This would happen, for instance, if the agent experiences a negative wage shock, a sudden improvement in the entrepreneurial technology or if the entrepreneurial idea arrives earlier than expected. In any of these cases, the agent might choose to invest in a risky project in order to reduce the instant decline in consumption that would

[26]See, for instance, Quadrini (1999), Bohacek (2006), Cagetti and DeNardi (2006), etc.

have to occur if he chooses to operate a safe project. The observation that the presence of uninsured risk can induce entrepreneurs to take excess risk is interesting per se. It suggests that the mechanism of risk taking studied in this paper also describes a possible channel of risk amplification[27] that may arise in the models of dynamic discrete choice. Whether or nor this channel is quantitatively significant remains an open question that should be studied separately.

E. Numerical Example

The above theoretical analysis predicts that risk taking can occur only for intermediate levels of β. Here we perform a series of numerical exercises with the goal of assessing whether entrepreneurial risk taking may arise and potentially be quantitatively significant under plausible parameter values. We use a CRRA utility function with relative risk aversion coefficient $\sigma = 2$. The time period is taken to be a year, so we set interest rate to 4 percent. Moskowitz and Vissing-Jorgensen (2002) report that the average return to entrepreneurial activity is about 10 percent, so we set $A = 1.10$. We simulate the model for different values of time discount factor β from the range $(1/A, 1/(1+r))$. Our results are not sensitive to the wage rate ϕ, so we fix $\phi = 1$.

Figure 5 characterizes the choice of entrepreneurial projects for $\beta = 0.958$. The left plot illustrates the occupational decision and depicts the survival probability p of the entrepreneurial projects chosen at corresponding wealth levels. Notice that for these parameter values, the risk taking interval is non-empty, and there are agents who choose to start as workers, accumulate wealth and eventually switch to "risky" entrepreneurial activity. Depending on the initial wealth level, it can take them up to twenty three years to become sufficiently rich and change their occupation. At the moment of entry into entrepreneurship, these agents invest in a risky project, which generates a successful

[27] In the existing literature, credit constraints have been typically suggested as a channel for risk amplification. See, for instance Nobuhiro Kiyotaki and John Moore (1997) and Cooley et. al. (2004).

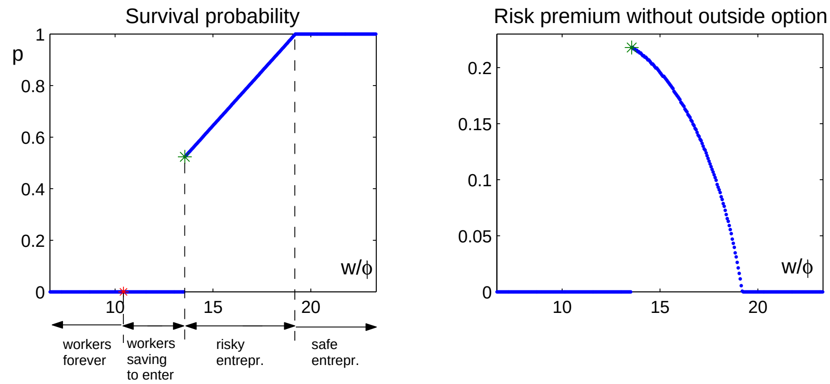


Figure 5: Optimal project choice and risk premium that would be required in the absence of outside opportunity; $r = 0.04,\ \beta = 0.958,\ A = 1.1,\ \sigma = 2$.

outcome with probability $p = 0.52$ (labeled by a star on the graph). This means that with 48 percent probability these new entrepreneurs exit from business in the following year. At the same time, conditional on staying in business, they obtain a project return of almost 60 percent (compared to the return of 10 percent to a safe project).

The standard deviation of returns of the entrant's risky project exceeds 50 percent (five times larger than its expected return). Such high volatility of the entrant's project is generated in our model due to the presence of the outside opportunity and does not require any risk premium. If the outside opportunity were not available, a risk premium would be necessary to justify such entrepreneurial risk taking. The right plot of Figure 5 illustrates that these risk premia could indeed be very high. In particular, in the absence of the outside opportunity, the poorest entrepreneur would be willing to invest in his risky project only if it offered a premium of 21 percent over a fully safe entrepreneurial return of 10 percent. For comparison, Heaton and Lucas (2004) argued, using a representative agent portfolio choice model, that, with $\sigma = 2$, entrepreneurs should be compensated with a premium of at least 10 percent in order to justify the high volatility of their returns observed in the data.

As we pointed out earlier, risk taking would not occur if, prior to the occupational

Table 1: The effect of β on risk taking incentives.

β	0.961	0.960	0.959	**0.958**	0.956	0.954	0.952
p	1	0.99	0.77	**0.52**	0.30	0.28	0.27
premium	0	0.01	0.16	**0.21**	0.19	0.18	0.17
max time to entry	61	54	40	**23**	3	1	0

decision, the agent could take a fair wealth lottery. In reality, however, fair wealth lotteries are hard to come by, which may make risk taking strictly dominant. For instance, in this numerical example entreprenurial risk would be strictly prefered to a lottery at any wealth level if the lottery costs 3.6 percent or more of the randomized wealth. Obviously, such cost would have to be even smaller if risky projects offered some premium over the risk-free projects.

Table 1 summarizes the effects of a change in agent's patience β on the entrant's project choice. The column in bold corresponds to the case of $\beta = 0.958$ illustrated in Figure 5. From Proposition 4 we know that no risk taking occurs when $\beta \geq 1/(1+r) \approx 0.9615$. Table 1 shows that the risk taking interval is empty even for slightly smaller values of β, namely for all $\beta \geq 0.961$, and in our numerical exercises risk taking occurs for all $\beta \leq 0.96$. Consistently with the discussion in the previous section, all entry into entrepreneurship happens immediately if the agents are sufficiently impatient ($\beta \leq 0.953$). However, for the intermediate patience levels ($\beta \in (0.953, 0.961)$), some agents postpone entering into entrepreneurship, choose to be workers in the beginning of their life, accumulate wealth and eventually enter as risky entrepreneurs. The last row of the table reports the number of periods that the poorest of these agents (with wealth $\widetilde{w}$) choose to wait before changing their occupation. At the moment of entry, they choose to operate a risky project and remain in business with probability p reported in the first row. As can be seen, these agents take quite a lot of risk – in the absence of the outside option they would invest in the projects with the same degree of risk only if they were offering premia of 15 percent and more (as reported in the second row of Table 1).

Do values of β for which entrepreneurial risk taking occurs seem plausible? From Proposition 4 it follows that $\beta(1 + r) < 1$ is a necessary condition for risk taking in our model. It is well known that this relationship between the rate of time preference and the interest rate endogenously arises in general equilibrium Bewley models with borrowing constraints and uninsured risk. For instance, in Rao S. Aiagari's (1994) calibrated model, $\beta(1 + r)$ varies from 0.93 to 0.999 for different parameters of the agents' labor income process. In our model, the risk taking interval is non-empty for all levels of β consistent with Aiagari's findings. Of course, such observation should be made with caution since there is no uninsured risk in our model. In general, the effects of uninsured risk on the amount of entrepreneurial risk taking are not obvious: risky projects may become more attractive for some realizations of uncertainty (as it was argued in the previous section), but less attractive for others. A natural extension of our model would be to add uninsured risk in our environment and analyze (perhaps, numerically) to what extent this would affect incentives for entrepreneurial risk taking.

There is one more obvious reason for incorporating uninsured risk in our model. Undoubtedly, the model developed in the paper is too stylized. For instance, the availability of a fully safe entrepreneurial technology and the absence of uncertainty in worker's income imply that in the long run the economy would be populated only by poor workers (with no wealth) and rich safe entrepreneurs. Thus our model cannot actually say anything about the amount of entrepreneurial risk taking that could occur at the aggregate level. In contrast, in a modified model with uninsured risk, there would exist a non-degenerate long run wealth distribution, implying endogenous separation of agents into workers, risky entrepreneurs and safe entrepreneurs. A model like this could then be used to assess whether the mechanism of entrepreneurial risk taking studied in this paper may have important quantitative implications at the aggregate level. One step in that direction is taken in a simple quantitative exercise described in online Appendix C (section 2.2). We find that, for particular processes of workers' wage and entrepreneurial returns, the amount of excess risk taking is quantitatively significant: almost 13 percent of all entrepreneurs decide to invest in excessively risky projects and, as a result, the standard deviation of observed returns to entrepreneurial activity rises from 4 percent to 12 percent.

III. Final Remarks

Entrepreneurship is risky, but there appears to be little (or no) premium to entrepreneurial activity. Any theory addressing this puzzle must rely, directly or indirectly, on a positive – or at least neutral – attitude towards risk. Earlier papers in this area assume directly that entrepreneurs have a lower degree of risk aversion. In our paper, the indirect utility function of the entrepreneur has a nonconcave region, where riskiness is desired. However, this nonconcavity is created by the possibility of discrete occupational choice, so it does not rely on assumptions about preferences for risk.

As a theory of risk taking, our model has specific implications. The combination of the outside option and financing constraints imply a desire for risk at low wealth levels, close to the exit threshold. As a consequence, risk taking decreases with the level of wealth, giving rise to the positive correlation between size (measured by investment) and survival found in the data. This is an implication of our theory that would be hard to derive just from the heterogeneity of preferences. As an example, Cressy (2000) justifies risk-taking by entrepreneurs assuming that higher wealth makes agents less risk averse. A consequence of this assumption is that larger firms should take more risk and thus exhibit more variable growth, which is counter to the data.

The simplicity of the characterization of entrepreneurial risk choice in our paper is driven by the fact that we allow entrepreneurs to invest in projects with two-point distribution of returns. Because of this assumption, entrepreneurs can eliminate all non-concavities in their continuation value by choosing the projects which put all weight at the tangent points of the concave envelope of the two value functions. To emphasize the robustness of our argument, it would be useful to investigate how the predictions of the model might be affected if we restrict attention to a narrower set of available projects. One step in this direction is taken in the online Appendix C-1, where we quantitatively analyze the implications of restricting the set of available projects by allowing entrepreneurs to invest only in projects with uniform returns distribution. We have found that the main qualitative properties of project choice obtained in our basic model (such as the relationship between entrepreneurial wealth, project survival and returns conditional on survival) are maintained even if we eliminate the projects with degenerate distribution of returns from the class of available projects.

Another shortcoming of our model is the absence of public firms, which prevents us from directly relating our results to the "private equity premium puzzle" documented by Moskowitz and Vissing-Jorgensen (2002). One way of introducing public equity in the model is by allowing firms to become public as they mature. There are problems with this approach. Given the assumptions of our technology (i.e. linearity of payoffs), a market for public equity would provide a perfect substitute for entrepreneurship: one public firm would suffice to take advantage of the superior technology. This can obviously be fixed by assuming decreasing returns to capital, so that public equity is in short supply. However, open participation in the stock market would drive the returns to public equity down, giving rise to a private equity premium. Thus our model does not provide a resolution to the "private equity premium puzzle" but it may contribute into narrowing the gap by providing a new explanation for why entrepreneurs are willing to take excessive risk. Potentially, it would be interesting to endogenize the supply of public equity and the relative public and private returns in order to evaluate how much of a private equity premium puzzle can be explained by our mechanism of risk taking.

IV. References

Albuquerque, Rui, and Hugo A. Hopenhayn. 2004. "Optimal Lending Contracts and Firm Dynamics." *The Review of Economic Studies*, 71(2): 285–314.

Akyol, Ahmet, and Kartik Athreya. 2008. "Unsecured Credit and Self-Employment." http://econ.yorku.ca/akyol/Uns_Cred_Self_Emp_April28.pdf.

Ayagari, S. Rao. 1995. "Uninsured Idiosyncratic Risk And Aggregate Saving." *Quarterly Journal of Economics*, 109(3): 659–684.

Blanchflower, David, and Oswald Andrew. 1998. "What Makes an Entrepreneur?" *Journal of Labor Economics*, 16(1): 26-20.

Bohacek, Radim. 2006. "Financial Constraints and Entrepreneurial Investment." *Journal of Monetary Economics,* 53(8): 2195-2212.

Buera, Francisco J. 2008. "Persistency of Poverty, Financial Frictions, and Entrepreneurship." http://faculty.wcas.northwestern.edu/ fjb913/paper120071217.pdf.

Campanale, Claudio. 2008. "An Exploration of Private Equity Returns." http://merlin.fae.ua.es/claudio/PEP.pdf.

Cagetti, Marco, and Mariacristina De Nardi. 2006. "Entrepreneurship, Frictions and Wealth." *Journal of Political Economy*, 114(5): 835-870.

Clementi, Gian L., and Hugo A. Hopenhayn. 2006. "A Theory of Financing Constraints and Firm Dynamics." *Quarterly Journal of Economics*, 121(1): 229–265.

Cooley, Thomas, Ramon Marimon, and Vincenzo Quadrini. 2004. "Aggregate Consequences of Limited Contract Enforceability." *Journal of Political Economy*, 112(4): 817-847.

Cressy, Robert. 2000. "Credit Rationing or Entrepreneurial Risk Aversion? An Alternative Explanation for the Evans and Jovanovic Finding." *Economic Letters*, 66(2): 235–240.

Davis, Steven, and John C. Haltiwanger. 1992. "Gross Job Creation, Gross Job Destruction, and Employment Reallocation." *Quarterly Journal of Economics*, 107(3): 819–63.

Dunne, Timothy, Mark J. Roberts and Larry Samuelson. 1989. "The Growth and Failure of US Manufacturing Plants." *Quarterly Journal of Economics*, 104(4): 671–698.

Ericson, Richard, and Pakes Ariel. 1995. "Markov Perfect Industry Dynamics: A Framework for Empirical Work." *Review of Economic Studies*, 62(1): 53–82.

Evans, David S. 1987. "Tests of Alternative Theories of Firm Growth." *Journal* *of Political Economy*, 95(4): 657–674.

Evans, David S., and Boyan Jovanovic. 1989. "An Estimated Model of Entrepreneurial Choice under Liquidity Constraints." *The Journal of Political Economy*, 97(4): 808–827.

Friedman, Milton, and Leonard J. Savage. 1948. "The Utility Analysis of Choices Involving Risk." *The Journal of Political Economy,* 56(4): 279-304.

Fazzari, Steven, R. Glenn Hubbard, Bruce C. Petersen, Alan S. Blinder, and James M Poterba. 1988. "Financing Constraints and Corporate Investment." *Brookings Papers on Economic Activity*, 1(1): 141–206.

Gentry, William M., and R. Glenn Hubbard. 2004. "Entrepreneurship and Household Saving." *Advances in Economic Analysis and Policy,* 4(1), Article 8, http://www.bepress.com/bejeap/advances/vol4/iss1/art8 .

Gertler, Mark, and Simon Gilchrist. 1994. "Monetary Policy, Business Cycles, and the Behavior of Small Manufacturing Firms." *Quarterly Journal of Economics*, 109(2): 309–340.

Gomes, Joao F. 2001. "Financing Investment." *American Economic Review*, 91(5): 1263–1285.

Greenwood, Jeremy, and Boyan Jovanovic. 1990. "Financial Development, Growth, and the Distribution of Income." *Journal of Political Economy,* 98(5): 1076-1107.

Hamilton, Barton H. 2000. "Does Entrepreneurship Pay? An Empirical Analysis of the Returns to Self-Employment." *Journal of Political Economy*, 108(3): 604–631.

Hansen, Gary D. 1985. "Indivisible Labor and the Business Cycle." *Journal of Monetary Economics*, 16(3): 309–327.

Heaton, John, and Deborah Lucas. 2000. "Portfolio Choice and Asset Prices: The Importance of Entrepreneurial Risk." *Journal of Finance*, 55(3): 1163–1198.

Heaton, John and Deborah Lucas. 2001. "Capital Structure, Hurdle Rates, And Portfolio Choice - Interactions in an Entrepreneurial Firm." http://www.fma.org/SLC/Papers/Lucas.pdf.

Herranz, Neus, Stefan Krasa, and Anne Villamil. 2007. "The Impact of Entrepreneur Characteristics and Bankruptcy Rules on Firm Performance." http://www.econ.uiuc.edu/ avillami/risk_aversion_May_2007.pdf.

Holtz-Eakin, Douglas, David Joulfaian, and Harvey S Rosen. 1994. "Sticking it Out: Entrepreneurial Survival and Liquidity Constraints." *Journal of Political Economy*, 102(1): 53–75.

Hopenhayn, Hugo A. 1992. "Entry, Exit, and Firm Dynamics in Long Run Equilibrium." *Econometrica*,60(5): 1127–1150.

Jovanovic, Boyan. 1982 "Selection and the Evolution of Industry." *Econometrica*, 50(3): 649–70.

Khan, Aubhik, and B. Ravikumar. 2002. "Costly Technology Adoption and Capital Accumulation." *Review of Economic Dynamics*, 5(2): 489–502.

Kihlstrom, Richard, and Jean-Jacques Laffont. 1979. "A General Equilibrium Entrepreneurial Theory of Firm Formation Based on Risk Aversion." *The Journal of Political Economy*, 87(4): 719–748.

Kiyotaki, Nobuhiro, and JohnMoore. 1997. "Credit Cycles." *Journal of Political Economy*, 105(2): 211-249.

Lucas, Robert E. 1978. "On the Size Distribution of Business Firms." *Bell Journal of Economics*, 9(2): 508–523.

Moskowitz, Tobias J., and Annette Vissing-Jorgensen. 2002. "The Returns to Entrepreneurial Investment: A Private Equity Premium Puzzle?" *American Economic Review*, 92(4): 745–778.

Paulson, Anna L., Robert M. Townsend, and Alexander Karaivanov. 2006. "Distinguishing Limited Liability from Moral Hazard in a Model of Entrepreneurship." *The Journal of Political Economy*, 114(1): 100–144.

Polkovnichenko, Valery. 2002. "Human Capital and the Private Equity Premium." *Review of Economic Dynamics,* 6(4): 831–845.

Quadrini, Vincenzo. 2000. "Entrepreneurship, Saving, and Social Mobility." *Review of Economic Dynamics,* 3(1): 1–40.

Rogerson, Richard. 1988. "Indivisible Labor, Lotteries and Equilibrium." *Journal of Monetary Economics,* 21(1): 3–16.

Appendix

Proof of Proposition 3:

If $\beta(1+r) \geq 1$, workers with the wealth sufficiently close to the threshold level w_E plan to become entrepreneurs in the following period. This means that the cutoff level of wealth w_E is determined by the intersection of the value function of an entrepreneur $V_E(w)$ and the value function of a worker $V_W^1(w)$ who plans to change occupation in the next period.[28] The latter value is found as follows:

$$V_W^1(w) = \max_{a \geq 0}\{u(w + \phi - a) + \beta V_E((1+r)a)\}. \tag{13}$$

Denote by w_H^1 the wealth level at which the worker solving the decision problem (13) saves exactly $\overline{w}/(1+r)$, i.e. $V_W^1(w_H^1) = u(w_H^1 + \phi - \overline{w}/(1+r)) + \beta V_E(\overline{w})$. By the first order condition, the current period consumption of this worker satisfies $u'(w_H^1 + \phi - \overline{w}/(1+r)) = \beta(1+r)V_E'(\overline{w})$.

Recall that the upper bound of the risk taking interval w_H corresponds to the wealth level at which a risk-free entrepreneur invests exactly $\overline{w}/A$ in his project, i.e. $V_E(w_H) = u(w_H - \overline{w}/A) + \beta V_E(\overline{w})$. Correspondingly, by the first order condition, $u'(w_H - \overline{w}/A) = \beta A V_E'(\overline{w})$. Since $1 + r < A$, it follows that $u'(w_H^1 + \phi - \overline{w}/(1+r)) < u'(w_H - \overline{w}/A)$ and, by concavity of $u(\cdot)$, this implies that $u(w_H^1 + \phi - \overline{w}/(1+r)) > u(w_H - \overline{w}/A)$. Thus the

[28]The rigorous proof of these statements is contained in the proof of Proposition 2 in the online appendix.

worker at w_H^1 enjoys higher current period consumption than the entrepreneur at w_H, while they have same continuation value $V_E(\overline{w})$. Thus $V_W^1(w_H^1) > V_E(w_H)$. Note that if $w_H^1 \leq w_E$ then it must be that $w_H \leq w_E$, thereby implying that risk taking entrepreneur do not exist.

We remain to verify that $w_H^1 \leq w_E$ indeed holds when $\beta(1+r) \geq 1$. This is done in two steps: (i) first, we sketch an argument establishing the exact position of $\underline{w}$ and $\overline{w}$, and (ii) second, we show that for such $\underline{w}$ and $\overline{w}$ the relationship $w_H^1 \leq w_E$ holds.

(i) Note that, in general, the value of the worker $V_W(w)$ can be represented as $V_W(w) = \max\{V_W^\infty(w), V_W^1(w), V_W^2(w), ...\}$, where $V_W^t(w)$ is the value of the worker who becomes entrepreneur in t periods, $t = 1, 2, ...$ or $t = +\infty$. It turns out that, when $\beta(1+r) \geq 1$, the common tangent line to $V_W(w)$ and $V_E(w)$ coincides with the common tangent line to $V_W^1(w)$ and $V_E(w)$. To see this, consider a worker who plans to become entrepreneur in two periods. His value $V_W^2(w)$ is given by

$$V_W^2(w) = \max_{a \geq 0}\{u(w + \phi - a) + \beta V_W^1((1+r)a)\}. \tag{14}$$

In the proof of Proposition 2 (see Lemma 2 in online Appendix B) we prove that $V_W^2(w)$ and $V_W^1(w)$ have a unique intersection which occurs to the left of w_E (which implies that the wealth of the workers who are planning to become entrepreneurs increases over time). Denote by $\underline{w}^1$ and $\overline{w}^1$ the tangent points of $V_W^2(w)$ and $V_W^1(w)$ respectively with their common tangent line (see Figure 6). They must satisfy the following relationship:

$$V_W^{2'}(\underline{w}^1) = V_W^{1'}(\overline{w}^1) = \frac{V_W^1(\overline{w}^1) - V_W^2(\underline{w}^1)}{\overline{w}^1 - \underline{w}^1}. \tag{15}$$

Invoking the first order and envelope conditions, as well as noticing that $V_W^{2'}(\underline{w}^1) = V_W^{1'}(\overline{w}^1)$ implies that the workers at $\underline{w}^1$ and $\overline{w}^1$ have the same current period consumption,

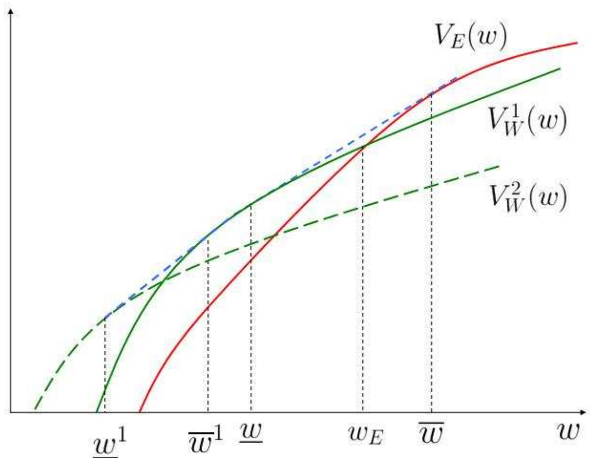


FIGURE 6: VALUES $V_E(w)$, $V_W^1(w)$ AND $V_W^2(w)$ FOR $\beta(1+r) \geq 1$

we obtain

(16)
$$V_W^{1'}((1+r)a_W(\underline{w}^1)) = V_E^{'}((1+r)a_W(\overline{w}^1)) = \frac{V_E((1+r)a_W(\overline{w}^1)) - V_W^1((1+r)a_W(\overline{w}^1))}{(1+r)a_W(\overline{w}^1) - (1+r)a_W(\overline{w}^1)}.$$

However, the above relationship can be satisfied only for the tangent points $\underline{w}$ and $\overline{w}$ of the value functions $V_W^1(w)$ and $V_E(w)$ with their common tangent line. Thus the workers at the tangent points $\underline{w}^1$ and $\overline{w}^1$ save exactly as much that their next period's wealth coincides with the tangent points $\underline{w}$ and $\overline{w}$ respectively. This observation is true for any value of $\beta(1+r)$. Next, by the first order conditions and envelope theorem, we obtain that

$$V_W^{1'}(\overline{w}^1) = \beta(1+r)V_E(\overline{w}) = \beta(1+r)V_W^{1'}(\underline{w}). \tag{17}$$

If $\beta(1+r) \geq 1$, (17) implies that $V_W^{1'}(\overline{w}^1) \geq V_W^{1'}(\underline{w})$, which, by concavity of $V_W^1(w)$ (see online Appendix), means that $\overline{w}^1 \leq \underline{w}$ (see Figure 6). Thus $\max\{V_W^1(w), V_W^2(w)\}$ lies below the common tangent line to $V_W^1(w)$ and $V_E(w)$. A straightforward inductive argument can be used to establish that the same property holds for $V_W(w) = \max\{V_W^1(w), V_W^2(w), ..., V_W^\infty(w)\}$.

(ii) Since $V_W^{1'}(\underline{w}) = V_E'(\overline{w})$ and $V_W^{1'}(w_H^1) = \beta(1+r)V_E'(\overline{w})$, it follows that $w_H^1 \leq \underline{w} < w_E$ as long as $\beta(1+r) \geq 1$. This implies that $w_E > w_H$, i.e. risky entrepreneurs do not exist.

Proof of Proposition 4:

We prove Proposition 4 in three steps: (i) first, we show that when β is sufficiently close to $1/A$, there are no workers who choose to save in order to eventually become safe entrepreneurs; then (ii) we establish that in this case the risk taking interval is non-empty; and, finally, (iii) we verify that for small β all entry into entrepreneurship happens at once.

(i) Suppose that $\beta = 1/A$. Recall that $V_E^f(w)$ defined in (**??**) corresponds to the value of the agent who remains a safe entrepreneur forever. Denote by $V_W^t(w)$ the value of the worker who plans to become a safe entrepreneur in t periods.

Note that value functions $V_W^1(w)$ and $V_E^f(w)$ have a unique intersection at w_0, and, since $\phi > 0$ and $1+r < A$, $V_W^1(w)$ is flatter than $V_E^f(w)$ at the intersection point.[29] Since $\beta = 1/A$, the safe entrepreneur with w_0 saves exactly w_0/A. At the same time, since the worker with w_0 enjoys higher instantaneous consumption than the entrepreneur and $V_W^1(w_0) = V_E^f(w_0)$, the worker at w_0 saves less than $w_0/(1+r)$, i.e. $(1+r)a_W^1(w_0) < w_0$.

It is then straightforward to verify that the worker at w_0 obtains higher value by postponing entry into entrepreneurship for one more period. The value of entering in two

[29]The first order and envelope conditions imply that $V_W^1(w) < V_E^f(w)$ at any w at which $(V_W^1(w))' \geq (V_E^f(w))'$.

periods is given by

$$V_W^2(w) = \max_{a\geq 0}\{u(w+\phi-a)+\beta V_W^1((1+r)a)\}, \tag{18}$$

while the value of entering in one period is given by

$$V_W^1(w) = \max_{a\geq 0}\{u(w+\phi-a)+\beta V_E^f((1+r)a)\}. \tag{19}$$

The optimal saving policy from (19) is feasible in (18), and, since $(1+r)a_w^1(w_0) < w_0$, such saving policy delivers strictly higher value in (18) than in (19). Thus the worker is strictly better off by postponing entry for two periods. By employing this argument inductively, we conclude that the worker should strictly prefer to remain a worker forever as opposed to saving and eventually becoming a safe entrepreneur. By continuity, the same conclusion pertains if β is sufficiently close to $1/A$.

(ii) Consider again the case of $\beta = 1/A$ (under the assumption that entrepreneurs do not start dissaving gradually in order to become workers in the future[30]). Suppose that no risk taking occurs. Then from (i) we know that the value of the agent is $\max\{V_W^\infty(w), V_E^f(w)\}$, where $V_W^\infty(w)$ and $V_E^f(w)$ have a unique intersection at some w^*, at which $V_W^\infty(w)$ is flatter than $V_E^f(w)$. Since $\beta A = 1$, the entrepreneur at w^* saves exactly w^*/A, which implies that he would be strictly better off investing in a risky project to eliminate the nonconcavity in his continuation value. As in (i), continuity implies that the same property holds for β sufficiently close to $1/A$.

(iii) In (ii) we established that for β close to $1/A$ risky entrepreneur must exist. Now we verify that all entry into entrepreneurship occurs at $t = 0$ when β is sufficiently small.

[30]When $\beta = 1/A$, such behavior can arise at some wealth levels. It, however, never occurs when $\beta > 1/A$. Since we are using the case of $\beta = 1/A$ only to make the limiting argument, we can abstract from the possibility of such behavior.

When $\beta = 1/A$, the value of the entrepreneur $V_E(w)$ coincides with the common tangent line to $V_W^{\infty}(w)$ and $V_E^f(w)$ in the interval $(\underline{w}, \overline{w})$ (where $\underline{w}$ and $\overline{w}$ are the tangent points), and with $V_E^f(w)$ for all $w \geq \overline{w}$. Notice that $\max\{V_W^{\infty}(w), V_E(w)\}$ is (weakly) concave. It is enough to verify that the value of becoming a worker and switching to entrepreneurship in the following period is strictly below $\max\{V_W^{\infty}(w), V_E(w)\}$. Denote by $\underline{w}_0$ the wealth level at which the worker maximizing $V_W^{\infty}(w)$ saves exactly $\underline{w}/(1+r)$. Since $\beta(1+r) < 1$, it must be that $\underline{w}_0 > \underline{w}$ and, hence, $V_W^{\infty}(\underline{w}_0) < V_E(\underline{w}_0)$. If the worker instead plans to become an entrepreneur in the following period, his value has unique intersections with both $V_W^{\infty}(w)$ and $V_E(w)$, cuts each of them from below, and must be equal to $V_W^{\infty}(\underline{w}_0)$ at $\underline{w}_0$. Correspondingly, such value must lie stricly below $\max\{V_W^{\infty}(w), V_E(w)\}$, which proves that all the entry into entrepreneurship happens at once. By continuity, the argument extends to the values of β close to $1/A$.